Exercise

1. a) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

b) If $\int_1^4 f(x)\,dx = 6$, find value of $\int_1^4 f(5-x)\,dx$

Solution

Part-I

Let $u = a + b - x$

Therefore, $du = -dx$

$x = a \Rightarrow u = b$

$x = b \Rightarrow u = a$

also, $x = a + b - u$

Notice that as x varies from a to b, u varies from b to a

$$\text{Thus, } \int_a^b f(x)\,dx = -\int_b^a f(a+b-u)\,du$$

$$= \int_a^b f(a+b-u)\,du$$

$$= \int_a^b f(a+b-x)\,dx$$

Note that since variable inside a definite integral is always a dummy variable, one can simply substitute u by x

Part-II

Note that we have $a = 1, \quad b = 4 \quad and \quad a + b - x = 5 - x$,

Therefore,

$$\int_1^4 f(5-x)\,dx = \int_1^4 f(1+4-x)\,dx$$

$$= \int_1^4 f(x)\,dx$$

$$= 6$$