Example

Show that $\lim_{n \to \infty} \frac{2^n}{n!} = 0$

Proof

$$2^n = (1+x)^n \text{ where } x = 1$$

$$= {}^nC_0 (x)^0 + {}^nC_1 (x)^1 + {}^nC_2 (x)^2 + \ldots + {}^nC_r (x)^r + \ldots + {}^nC_n x^n$$

$$= \sum_{r=0}^{n} x^r \, {}^nC_r$$

$$= \sum_{r=0}^{n} x^r \left(\frac{n!}{r!(n-r)!} \right)$$

$$= n! \sum_{r=0}^{n} x^r \left(\frac{1}{r!(n-r)!} \right)$$

Now substitute back $x = 1$

$$\frac{2^n}{n!} = \sum_{r=0}^{n} (1)^r \left(\frac{1}{r!(n-r)!} \right)$$

Taking limit as $n \to \infty$

$$\lim_{n \to \infty} \frac{2^n}{n!} = 0$$