

Q 1. Find $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x}$

(Trig limit at infinity)

Solution

On substituting $x = \infty$, $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x} = \frac{\infty + \text{something} \in [-1, 1]}{\infty + \text{something} \in [-1, 1]} = \frac{\infty}{\infty}$ which is indeterminate.

Hence

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x} &= \lim_{x \rightarrow \infty} \frac{1 + \frac{\sin x}{x}}{1 + \frac{\cos x}{x}} \\ &= \frac{1 + \lim_{x \rightarrow \infty} \frac{\sin x}{x}}{1 + \lim_{x \rightarrow \infty} \frac{\cos x}{x}} \\ &= \frac{1 + 0}{1 + 0} \\ &= 1 \end{aligned}$$

Note: For any x
 $-1 \leq \sin x$ and $\cos x \leq 1$
 This is not a $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ limit

Answer