Q 1. Find $\frac{dy}{dx}$ *for* $x^2 = \frac{x-y}{x+y}$ by implicit method of differentiation. Obtain an explicit function for y and f ind $\frac{dy}{dx}$. Are the values obatined for $\frac{dy}{dx}$ by two methods the same?

Solution

Method I

Differentiitng both sides of $x^2 = \frac{x-y}{x+y}$ wrt x, we have

$$2x(1) = \frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2}$$

$$\Rightarrow \quad 2x(x+y)^2 = (x+y) - (x+y)\frac{dy}{dx} - (x-y) - (x-y)\frac{dy}{dx}$$

$$\Rightarrow \quad 2x(x+y)^2 = (x+y-x+y) - \frac{dy}{dx}(x+y+x-y)$$

$$\Rightarrow \quad 2x(x+y)^2 = (2y) - \frac{dy}{dx}(2x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2y-2x(x+y)^2}{2x} = \frac{y-x(x+y)^2}{x} = \frac{y-x^3-2x^2y-xy^2}{x} \qquad (1)$$

<u>Answer</u>

Method II

Given $x^2 = \frac{x-y}{x+y}$

$$\Rightarrow \quad (x+y)x^2 = x-y$$

$$\Rightarrow \quad x^3 + x^2y = x-y$$

$$\Rightarrow \quad y(x^2+1) = x-x^3$$

$$\Rightarrow \quad y = \frac{x-x^3}{x^2+1} \qquad (2)$$

Differentiating wrt x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(x^2+1)(1-3x^2)-(x-x^3)(2x)}{(x^2+1)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2-3x^4+1-3x^2-2x^2+2x^4}{(x^2+1)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x^4-4x^2+1}{(x^2+1)^2} \qquad (3)$$

<u>Answer</u>

Values for $\frac{dy}{dx}$ by implicit and explicit differentiation:

Answers (1) and (3) are same and can be easily verified if value of y from (2) is substituted in (1).