

Q 1. Prove that $\frac{\tan x + \sec x - 1}{\tan x - \sec x + 1} = \frac{1 + \sin x}{\cos x}$

[Trig identity]

Solution

$$\begin{aligned}
 \text{LHS} &= \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1} &&= \frac{\tan x + \sec x + [-1]}{\tan x - \sec x + 1} \\
 &&&= \frac{\tan x + \sec x + [\tan^2 x - \sec^2 x]}{\tan x - \sec x + 1} \\
 &&&= \frac{\tan x + \sec x + [(\tan x + \sec x)(\tan x - \sec x)]}{\tan x - \sec x + 1} \\
 &&&= \frac{(\tan x + \sec x)(1 + \tan x - \sec x)}{\tan x - \sec x + 1} \\
 &&&= \tan x + \sec x \\
 &&&= \frac{\sin x}{\cos x} + \frac{1}{\cos x} \\
 &&&= \frac{\sin x + 1}{\cos x} = \text{RHS}
 \end{aligned}$$

Recall:

$$\begin{aligned}
 \sec^2 x &= 1 + \tan^2 x \\
 \tan^2 x - \sec^2 x &= [-1]
 \end{aligned}$$

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QED